can imagine each simulation as a point in a 240 dimensional hypercube. LDS is a mathematically valid way to pick points in this 240 dimensional hypercube. It allows accurate calculation of tail measures, such as Value at Risk or CTE. LDS is also known as Quasi-Random Monte Carlo (QRMC).

Discrepancy is a term from theory of integration. A partition of points used to calculate an integral for a given function leads to a measure called discrepancy. The bound on the error of calculating the integral of CTE from a simulation is determined by the discrepancy measure. This is part of the theory of integration. The error of a partition sum for an integral is less than the variation of the function on the interval times the star discrepancy. This is the Koksma-Hlawka inequality. For multiple dimensions interval means hypercube.

There exist functions within epsilon of this error bound. Thus, the name of the game in integration is to reduce the discrepancy. The variation can't be changed because it's determined by the function being integrated and the interval. Thus the only way to reduce the error bound independent of the function it is applied to is to reduce the discrepancy. This is a theorem in theory of integration.

The Finder system produces sequences of random variables that are either normally distributed or are uniform deviates. These sequences can be used in the MFC ESG or can be used in another simulation system. The Finder module is a stand-alone module and can be licensed separately from the ESG.

MFC Founder

Mathematical Finance Company™ (**MFC**™) was founded by Mark S. Tenney, who is currently president. He developed the **Double Mean Reverting Process**™ and the calculation engine **of the Economic Scenario Generator**™.

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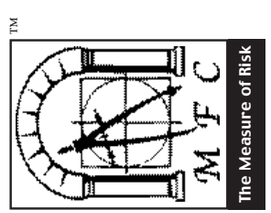

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™

A Beacon in the Sea of Risk

Mathematical Finance Company

MATHEMATICAL FINANCE COMPANY

ECONOMIC SCENARIO GENERATOR™ (ESG™)

Product Description

The Mathematical Finance Company (MFC) Economic Scenario Generator™ (ESG™) is a software program for producing scenarios of yield curves, equity indices, dividend yields, exchange rates and other series. It is based on the Double Mean Reverting Process™ (DMRP™). The DMRP process is embedded into a vector autoregression(VAR). This VAR jointly models yield curves, equities and other variables. The VAR can include multiple economies. Regime switching can be used for the VAR.

The model allows for multiple equity-like indices as part of the VAR. These can be used to model multiple equity portfolios, but also asset classes such as real estate or a foreign asset portfolio that is modeled on a return basis. By including a dividend series for these types of asset classes the system can produce both an income and total return value. The equity like indices can be correlated to each other and to the interest rate variables of the DMRP model.

Interest Rate Model Description

The DMRP interest rate model is a 2-factor interest rate model for the stochastic process on the yield curve. The DMRP models the logarithm of the short-term rate directly. It models this logarithm as being subject to a random shock but headed towards a target rate. This target rate is itself moving towards a long-term target rate, but is also subject to a random shock. The random shock on the target rate and on the logarithm of the short-term rate are correlated. The parameters of the model are the volatilities of the two shocks, the correlation of the two shocks, the two rates of mean reversion and the ultimate target rate that the target rate is headed towards.

The DMRP model has been calibrated to historical yield curves in both the U.S. economy and to the Canadian, Japanese, and Swiss economies. The model has been found to be robust over these multiple economies without dramatic alterations in its parameter values. The underlying logic of these values is that the rate of mean reversion of the logarithm of the short term rate is high towards the target rate, but the rate of mean reversion of the target rate to the ultimate target rate is low. Because of this, the target rate determines a trading range or a regime of the economy. The regime changes slowly towards the long run tendency of the economy. Thus long episodes of high, medium or low interest rates can occur in the model. The volatility of the target rate is lower than that of the logarithm of the short term interest rate so that a regime can persist. However, within a regime there can still be variation in rates.

The yield curve is determined at any point in time by the so-called risk neutral values of the parameters. In the risk neutral process, the target rate is higher than in the normal, or real probability process. Also the ultimate target rate is higher. These higher values reflect the presence of risk aversion against higher rates. The risk neutral process incorporates risk aversion by making higher rates more likely to occur. In addition, the rate of mean reversion of the logarithm of the short term rate towards the target rate is higher in the risk neutral process, while the rate of mean reversion of the target rate towards the ultimate target rate is lower. This reflects the historical shapes of yield curves observed. The higher rate of mean reversion of the logarithm of the short-term rate to the target rate means that the target rate has a bigger impact on the shape of the yield curve. This allows the yield curve to be more steeply slowed. The lower rate of mean reversion of the target rate to the ultimate target rate results in more allowed variation in the value of long-term yields. The two volatility parameters and the correlation parameters are the same from the risk neutral to the real probability.

By adjusting the parameter values, the different major industrial economies can be modeled. For example, Swiss interest rates have had a smaller variation in range historically than those in the U.S. This fact can be accommodated by primarily adjusting the rates of mean reversion. In particular, the rate of mean reversion of the target rate to the ultimate target rate is made higher. This keeps the range of rates more compact. Because interest rates are lower in the model, the absolute volatility automatically becomes lower with the lower absolute level of rates. This is because the logarithm of the interest rate is modeled, not the absolute level itself. Thus a given numerical value of the random shock results in a smaller change in interest rates when the absolute level of rates is smaller.

What the System Produces

The system produces two scenario files. The merged yield file contains scenarios of the two state variables of the interest rate model and yield curves. The other file contains the vector autoregression including the two state variables of the interest rate model, the equity indices and the dividend yield. These two files are ASCII files with spaces between the variables. They are in the following format.

scenario index, time index, short rate, target rate, y_1, y_2.,...,y_n

where y_1 to y_n are the yields of bonds in the merged yield file, or are the value of equity indices and the associated dividend yields in the other file. The system also has some conversion utilities to prepare these output files in other formats.

User Provided Input

The user provides the following input.

Points on the yield curve that are simulated and fit.
Starting value of the treasury yield curve for these points.
Number of scenarios
Time interval in scenarios

These values are entered in a screen interface.

The user then launches the calculation from the interface after having selected these values.

In addition, the user can select different parameter files from the interface corresponding to different parameter sets for the interest rate model parameters. Examples are the real process parameters, the risk neutral and the parameters for different economies or different sub-periods. Pre-packaged parameters for different economies or currencies must be purchased separately at a modest fee above the base fee for the system. The user can modify these or enter their own parameter sets.

FINDER

Low Discrepancy Sequences

Simulations in finance are high dimensional integrations. Each random variable at each time point counts as a separate dimension. So 2 variables quarterly for 30 years is 240 dimensions. If these were the sole random drivers of a simulation, then statistics from the simulation are 240 dimensional integrals. We